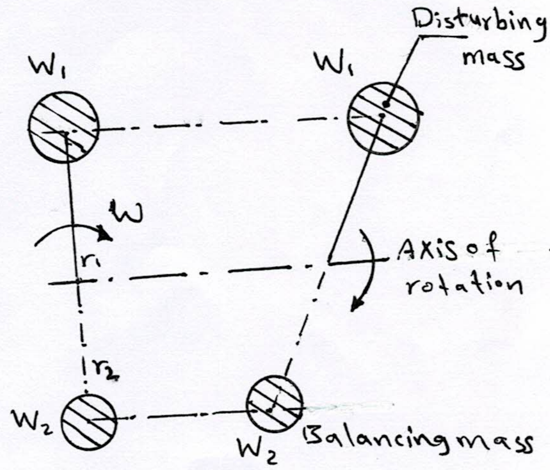
6- Balancing of Rotating Masses:

The high speed of engines and other machines is a common phenomenon now-a-days. It is, therefore, very essential that all the rotating and reciprocating parts should be completely balanced as far as possible. If these parts are not properly balanced, the dynamic forces are set up. These forces not only increase the loads on bearings and stresses in the various members, but also produce unpleasant and even dangerous vibrations.

6.1: Balancing of a single rotating mass by a single mass rotating in the same plane:

Consider a disturbing mass, of weight W_1, attached to a shaft rotating at w rad/sec. as shown in Figure. Let r_1 the radius of rotation of (W_1) i.e. the distance between the axis of rotation of the shaft and the center of gravity of the weight W_1.



The centrifugal force exerted by the weight

* W_1 on the shaft, $F_{c_1} = \frac{W_1}{g} w^2 r_1$ —①

∴ Centrifugal force due to weight W_2,

$$F_{c_2} = \frac{W_2}{g} w^2 r_2 \quad —②$$

Equating equations ① & ②

$$\frac{W_1}{g} w^2 r_1 = \frac{W_2}{g} w^2 r_2$$

$$W_1 r_1 = W_2 r_2$$

6.2. Balancing of Several Masses Rotating in the Same plane.

Let a four masses of wieghts, W_1, W_2, W_3 and W_4 at distances of r_1, r_2, r_3 and r_4 from the axis of the rotating shaft.

Let $\theta_1, \theta_2, \theta_3$ a[illegible] θ_4 be the angles of these masses with the horizontal OX. Let these masses rotate about an axis through O and perpendicular to the plane of paper with a constant angular velocity of ω rad/sec.

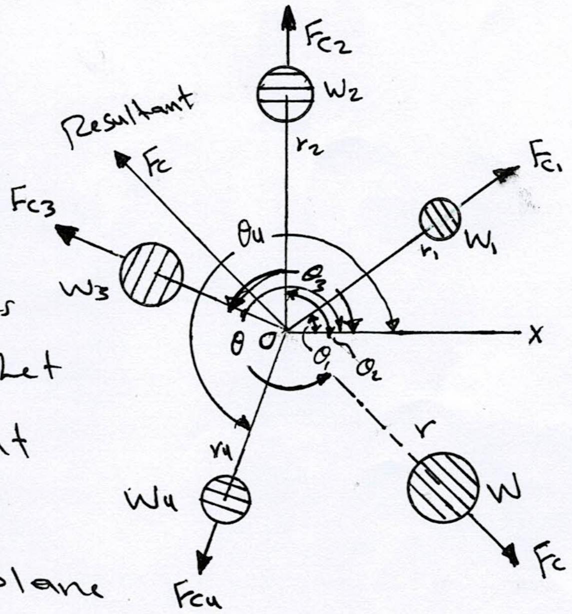


Fig.(a)

The magnitude and position of the balancing mass may be found out analytically or graphically as discussed below:

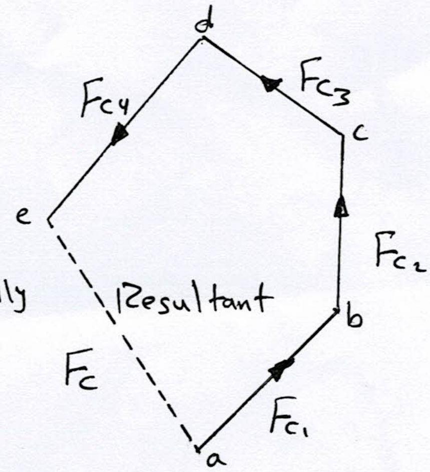


Fig.(b)

$* \sum H = W_1 r_1 \cos\theta_1 + W_2 r_2 \cos\theta_2 + \cdots$

Sum of horizontal components of centrifugal forces.

$** \sum V = W_1 r_1 \sin\theta_1 + W_2 r_2 \sin\theta_2 + \cdots$

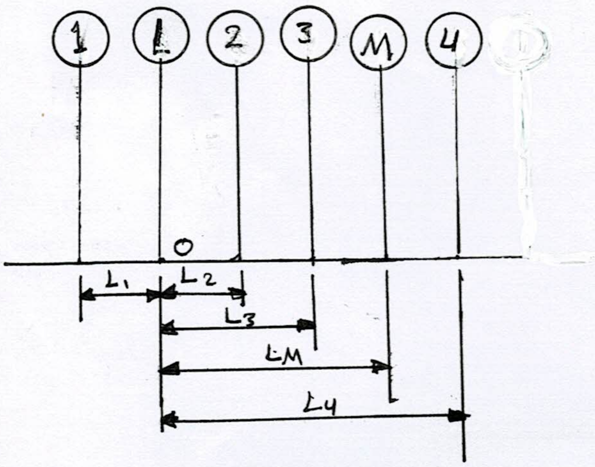
$$F_c = \sqrt{(\sum H)^2 + (\sum V)^2}$$ --- resultant Centrifugal force

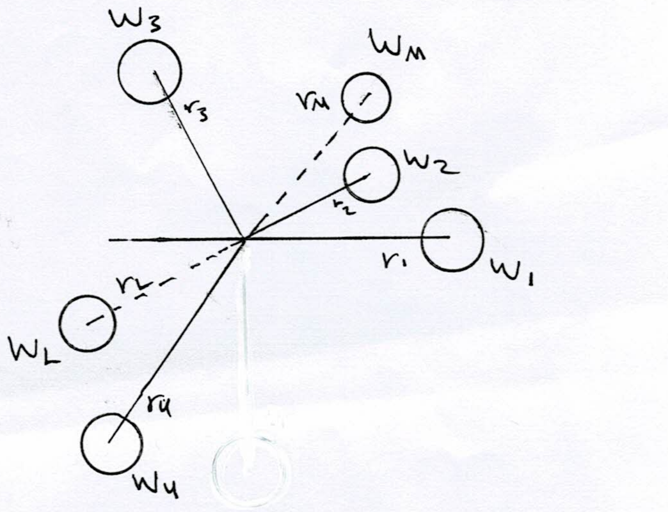
$$\tan\theta = \frac{\sum V}{\sum H}$$, θ: the angle which the resultant force makes with the horizontal.

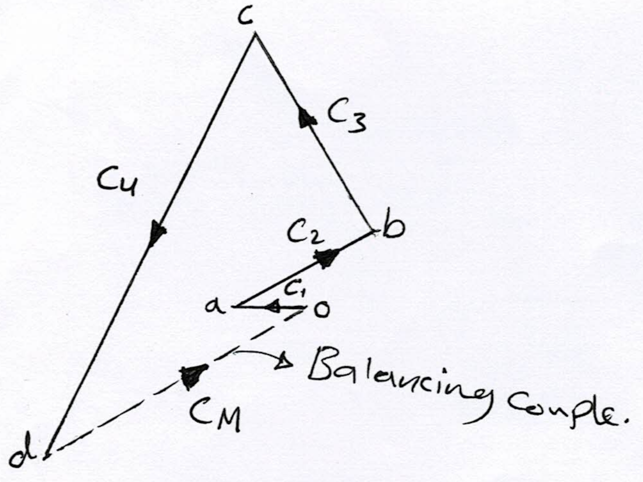
$$F_c = \frac{W}{g}\omega^2 r$$, W: weight of the balancing mass
r: Its radius of rotation.

6.3. Balancing of Several Masses Rotating in Different planes:—

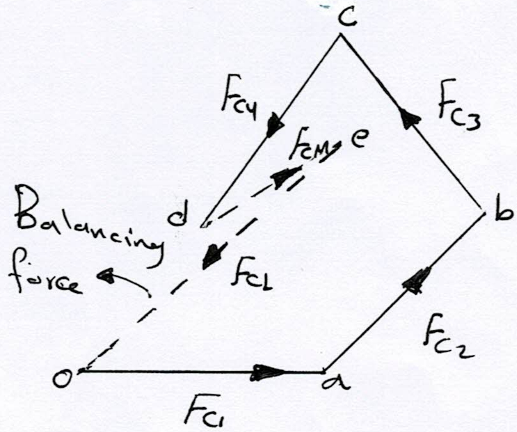
When several masses revolve in different planes, they may be transferred to a reference plane, which may be defined as the plane passing through a point on the axis of rotation and perpendicular to it. The effect of transferring a revolving mass (in one plane) to a reference plane is to cause a force of magnitude equal to the centrifugal force of the revolving mass to act in the refernce plane, together with a couple of magnitude equal to the product of the force and the distance between the plane of rotation and refrence plane.







Couple polygon

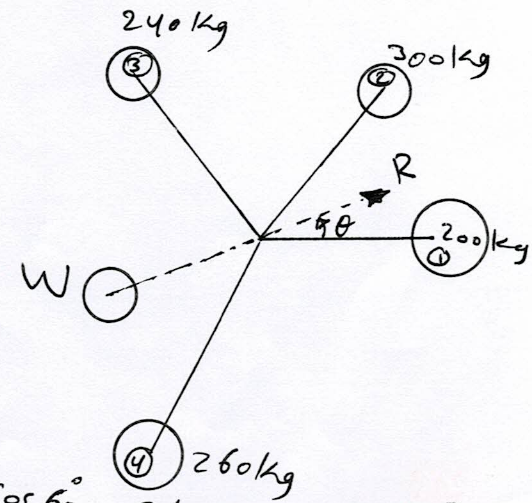


Force polygon.

EX.1:- The weights of four masses W_1, W_2, W_3 and W_4 are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radii of rotation are 20 cm, 15 cm, 25 cm and 30 cm respectively and the angles between successive masses are 45°, 75° and 135°. Find the position and magnitude of the balance weight required, if its radius of rotation is 20 cm.

Sol Let W = Balancing weight

θ = Angle of (B.W.) makes with W_1



1- Analytical method:

$\Sigma H = W_1 r_1 \cos\theta_1 + W_2 r_2 \cos\theta_2 + W_3 r_3 \cos\theta_3 + W_4 r_4 \cos\theta_4$

$= 200 \times 20 \cos 0° + 300 \times 15 \cos 45° - 240 \times 25 \cos 60° - 260 \times 30 \times \cos 75°$

$= 2122$ kg cm.

$\Sigma V = 200 \times 20 \times \sin 0° + 300 \times 15 \sin 45° + 240 \times 25 \sin 60° - 260 \times 30 \sin 75°$

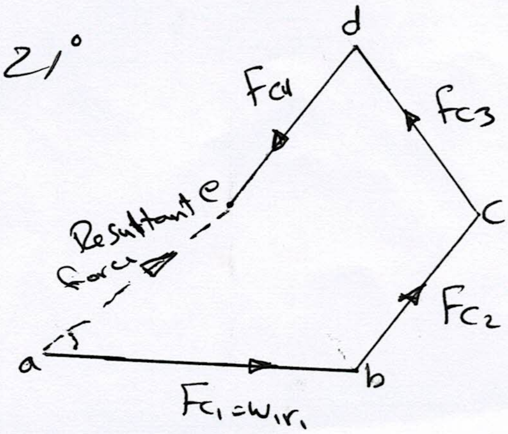
$= 811$ kg cm.

Resultant $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = 2200$ kg cm

$W \cdot r = R \Rightarrow W = \frac{2200}{20} = 110$ kg

$\tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{811}{2122} \Rightarrow \theta = 21°$

2. Graphical method:



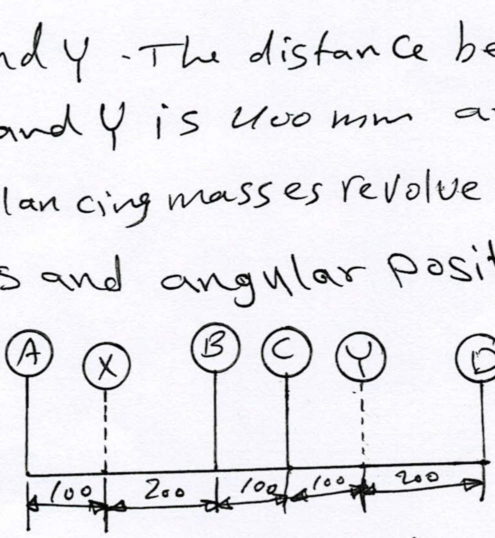
Draw the force polygon.

$F_c = Wr$ = vector ea = 2200 kg cm

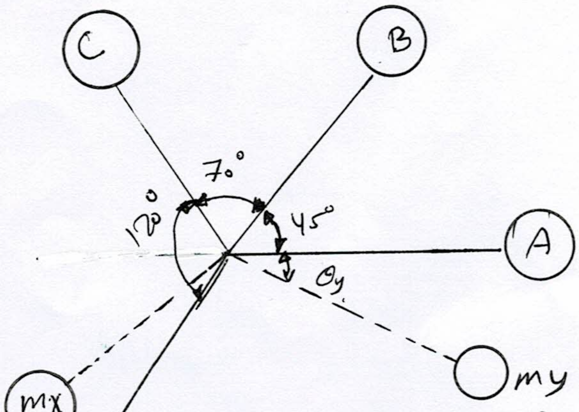
$\Rightarrow W = \frac{2200}{20} = 110$ kg

EX•2:- A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A to B 45°, B to C 70° and C to D 120°. The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100 mm, beteen X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions.

Solution:

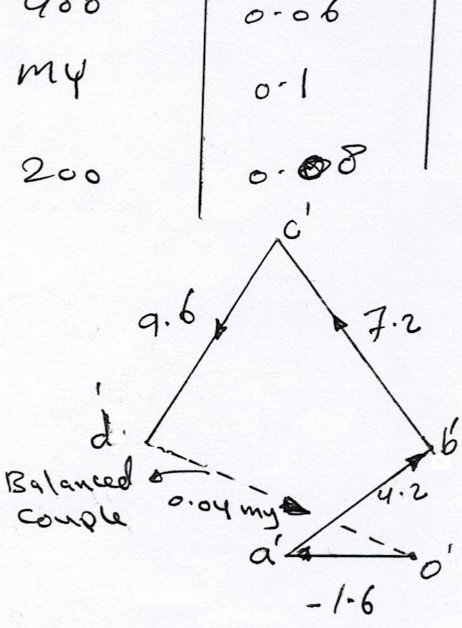


(a) Positions of planes



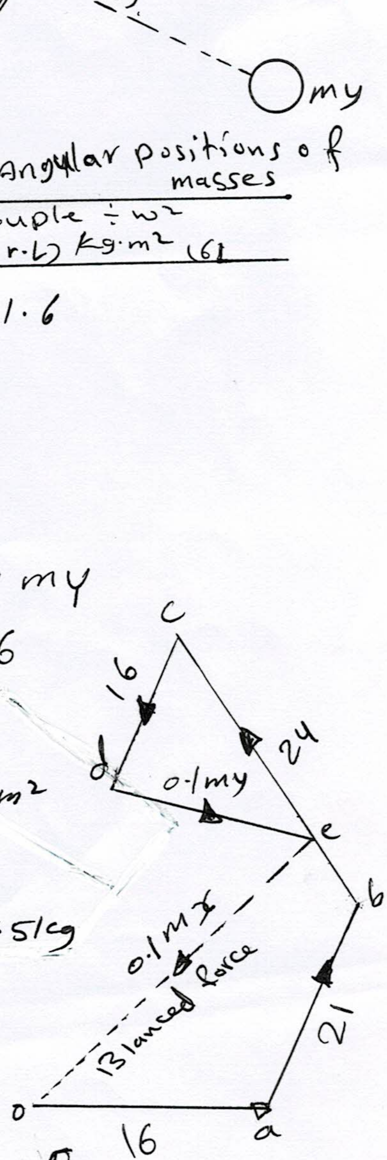
Angular positions of masses

Plane (1)	Mass (m) kg (2)	Radius (r) m (3)	Cet. force ÷ w2 (m.r) kg.m (4)	Distance (L) m (5)	Couple ÷ w² (m.r.L) kg.m² (6)
A	200	0.08	16	-0.1	-1.6
X (R.P.)	mx	0.1	0.1 mx	0	0
B	300	0.07	21	0.2	4.2
C	400	0.06	24	0.3	7.2
Y	my	0.1	0.1 my	0.4	0.04 my
D	200	0.08	16	0.6	9.6



Couple polygon

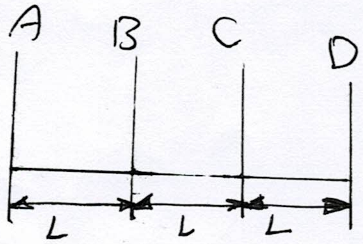
from couple polygon
$0.04 my = \text{vector } d'o' = 7.3 \text{ kg·m}^2$
$\Rightarrow my = 182.5$ kg
from force polygo
$0.1 mx = \text{vector } eo = 35.5$ kg
$\Rightarrow mx = 355$ kg.

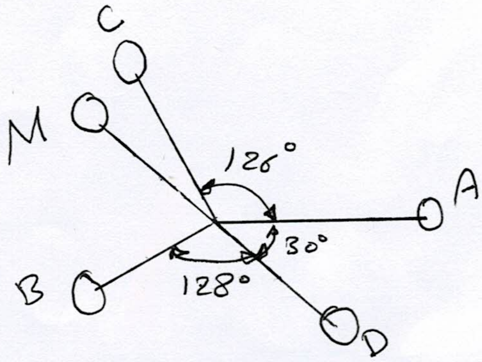


Force polygon

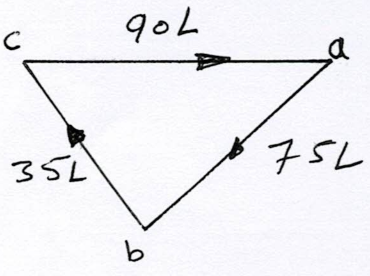
Ex.3 Attached to a uniformly rotating shaft are four discs, A, B, C and D, spaced at equal intervals along the shaft, of mass 7.5 Kg, 12.5 Kg, 7 Kg and 6 Kg respectively, the centres of mass of the discs are 4mm, 3mm, 5mm and 8mm respectively from the axis of rotation. An additional mass M may be attached to D at an effective radius of 60mm. Find the minimum value of the mass of M, and the relative angular positions of the centers of mass of all masses to ensure complete dynamic balance for the rotating shaft.

Sol.

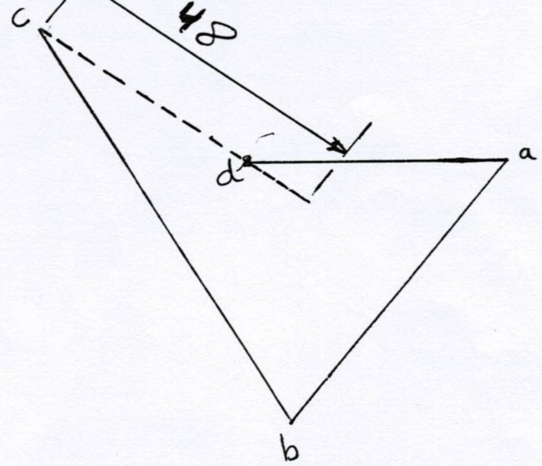




Plane	m	R	m·R	L	m·RL
A	7.5	4	30	3L	90L
B	12.5	3	37.5	2L	75L
C	7	5	35	L	35L
D	6	8	48	0	0
	M	60	60M	0	0



Couple polygon



Force polygon

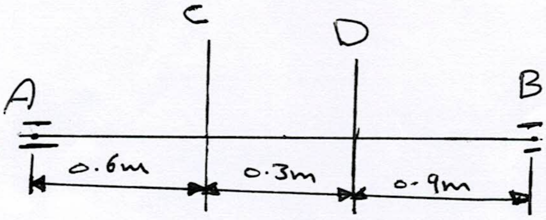
from force polygon

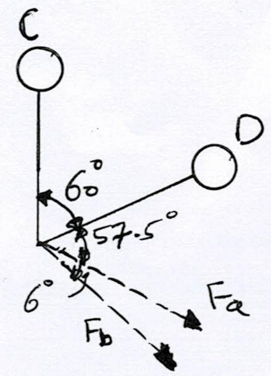
$48 + 60M = cd = 29.6$ Kg.mm

$\Rightarrow M = -0.307$ Kg

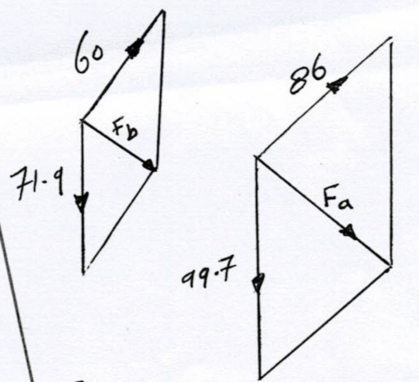
Ex.4:— A shaft rotating at 120 r.p.m. is supported in bearings A and B, 1.8 apart, at the left hand end. Two unbalanced rotating masses of 7.5 kg and 10 kg at radii of 75 mm and 50 mm respectively are situated between A and B at distances of 0.6 m and 0.9 m respectively from A. The angle between the radii is 60° when viewed along the shaft.
Find the magnitudes, directions and senses of the forces of the shaft on the bearings due to the combined action of the dynamical forces and gravity whe[n] the 7.5 kg mass is vertical and above the shaft and the 10 kg mass is on the right of the 7.5 kg mass when viewed from B to A. Show the results in an end view looking from B to A.

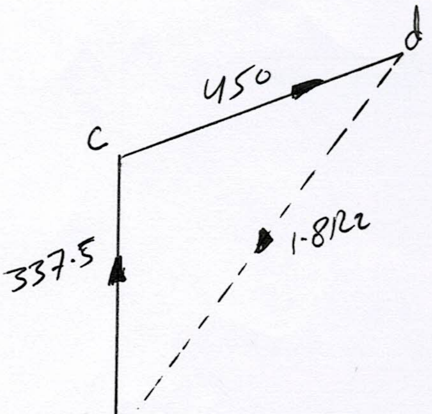
Sol



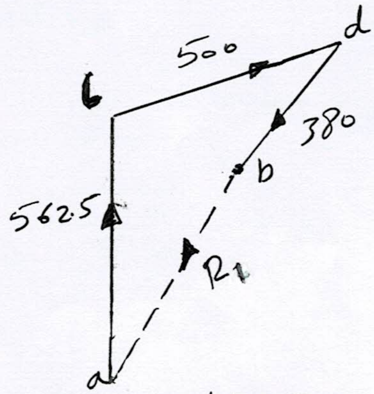


plane	m (kg)	r (mm)	mr	L (m)	m.r.L
A	—	—	R_1	0	0
B	—	—	R_2	1.8	$1.8R_2$
C	7.5	75	562.5	0.6	337.5
D	10	50	500	0.9	450





Couple polygon



Force polygon

from couple polygon $\Rightarrow 1.8R_2 = 685$ kg.mm.m

$\Rightarrow R_2 = 380$ kg.mm

dynamic force at $B = R_2 w^2 = \frac{380}{1000}\left(\frac{2\pi}{60} \times 120\right)^2 = \underline{\underline{60N}}$

from force polygon

$\Rightarrow R_1 = 545$ kg.mm

dynamic force at $A = \frac{545}{100}\left(\frac{2\pi}{60}120\right)^2 = 86N$

Force on bearing B due to dead weight $= \frac{7.5 \times 0.6 + 10 \times 0.9}{1.8} \times 9.81$

$= 71.9N \downarrow$

Force on bearing $A = 17.5 \times 9.81 - 71.9$

$= 99.7N \downarrow$

resultant force at $B = f_b = 36N$

" " at $A: f_a = 39.2N$

Theory of Machiner ((Balancing))

Q₁:- Four masses A, B, C and D revolve at equal radii and are equally spaced along a shaft. The mass B is 7 kg and the radii of C and D make angles of 90° and 240° respectively with the radius of B. Find the magnitude of the masses A, C and D and the angular position of A so that the system may be completely balanced.

[Ans. 5 kg; 6 kg; 4.67 kg; 205° from mass B in anticlockwise]

Q₂:- A, B, C and D are four masses carried by a rotating shaft at radii 100 mm, 150 mm, 150 mm and 200 mm respectively. The planes in which the masses rotate are spaced at 500 mm apart and the magnitude of the masses B, C and D are 9 kg, 5 kg and 4 kg respectively. Find the required mass A and the relative angular setting of the four masses so that the shaft shall be in complete balance.

[Ans. 10 kg, B to A 165°, B to C 295°, B to D 145°]

Q₃:- A shaft is supported in bearings 1.8 m apart and projects 0.45 m beyond bearings at each end. The shaft carries three pulleys one at each end and one at the middle of its length. The mass of end pulleys is 48 kg and 20 kg and their center of gravity are 15 mm and 12.5 mm respectively from the shaft axis. The center pulley has a mass of 56 kg and its center of gravity is 15 mm from the shaft axis. If pullyes are arranged so as to give static balance, determine:

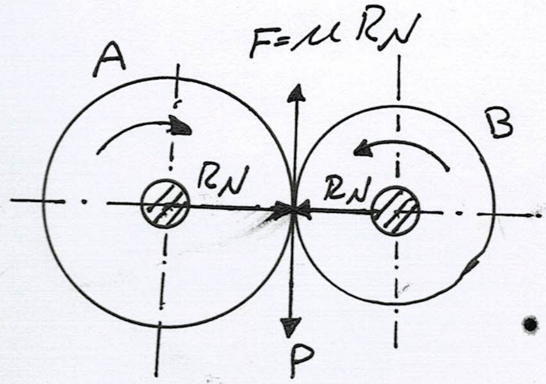
1- relative angular positions of the pullyes
2 dynamic forces produced on the bearings when the shaft rotates at 300 r.p.m.

[Ans. B to A 161°, A to C 76°, C to B 123°, 533 N, 533 N]

7. Spur gearing :-

7.1 Friction wheels: The motion and power transmitted by gears is kinematically equivalent to the transmitted by friction wheels or discs, consider two plain circular wheels A and B mounted on shafts, having sufficient rough surfaces.

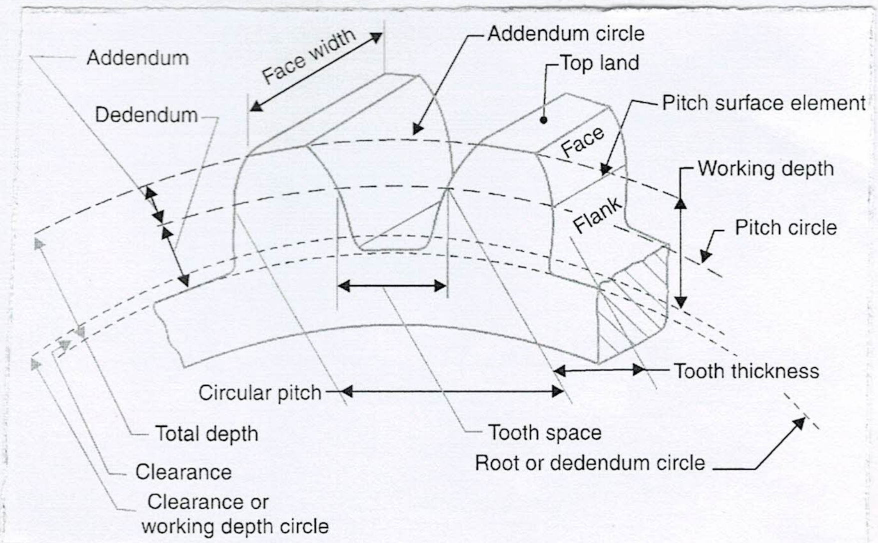
The wheel B will be rotated by the wheel A so long as $P \leq F$
But when $P > F$ slipping occurs.
F = Frictional resistance.
P = Tangential force.



In order to avoid the slipping, a number of projections (called teeth) are provided on the periphery of the wheel A, which will fit into the corresponding recesses on the periphery of the wheel B.

7.2 Terms used in Gears:

1. pitch circle: It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.



2. Pitch point: It is a common point of contact between two pitch circles.
3. Pressure angle: It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. Denoted ϕ, Standrad $14.5°$ and $20°$.

4. Circular pitch (P_c): It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth.

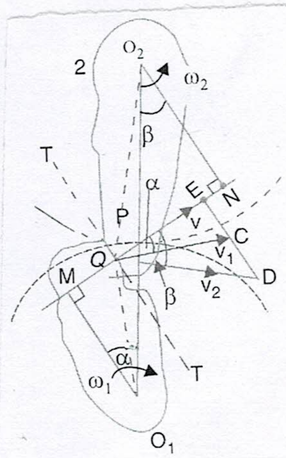
$$P_c = \pi D / T$$

If D_1 and D_2 are the diameters of the two meshing gears having the teeth T_1 and T_2, then for them to mesh correctly

$$P_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} \text{ or } \frac{D_1}{D_2} = \frac{T_1}{T_2}, \quad P_d = \frac{T}{D} = \frac{\pi}{P_c}, \quad P_d = \text{diametral pitch.}$$

5. Module (m): It is the ratio of the pitch circle diameter in millimeters to the number of teeth. $m = \frac{D}{T}$.

7-3. Condition for Constant Velocity Ratio of Toothed Wheels:



Let the two teeth come in contact at point Q, and the wheels rotates in the directions as show. MN be the common normal to the curves at the point of contact Q. A little consideration will show that the point Q moves in the direction QC, when considered as a point on wheel 1, and in the direction QD when considered as a point on wheel 2.

Let v_1, v_2 the velocities of the point Q on the wheels 1 and 2. If the teeth are to remain in contact, then the components of these velocities along the common normal MN must be equal.

$$v_1 \cos\alpha = v_2 \cos\beta$$

$$(\omega_1 \, O_1Q) \cdot \cos\alpha = (\omega_2 \, O_2Q) \cos\beta$$

$$\omega_1 \, O_1Q \, \frac{O_1M}{O_1Q} = \omega_2 \, O_2Q \, \frac{O_2N}{O_2Q}$$

$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M}$$

From similar triangles O_1MP and O_2NP.

$$\frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} \Rightarrow \frac{w_1}{w_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P}$$

If D_1 = diameter of wheel 1

D_2 = " " " 2 $\Rightarrow \frac{w_1}{w_2} = \frac{D_2}{D_1} = \frac{T_2}{T_1}$

* Velocity of Sliding of Teeth (v_s): It is the velocity of one tooth relative to its mating tooth along the common tangent at the point of contact.

The velocity of point Q, cosidered as a point on wheel 1, along the common tangent TT is represented by EC. From similar triangles QEC and O_1MQ

$$\frac{EC}{MQ} = \frac{v_1}{O_1Q} = w_1 \text{ or } EC = w_1 \cdot MQ$$

Similarly velocity of point Q, considered as a point on wheel 2, represented by ED. From similar triangles QED and O_2NQ.

$$\frac{ED}{QN} = \frac{v_2}{O_2Q} = w_2 \quad \text{or } ED = w_2 QN$$

Let v_s = Velocity of Sliding at Q.

$$v_s = ED - EC = w_2 QN - w_1 MQ$$

$$= w_2 (QP + PN) - w_1 (MP - QP)$$

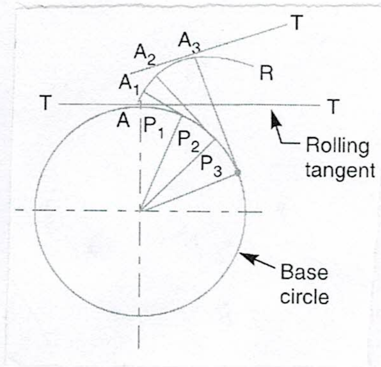
$$= (w_1 + w_2) QP + w_2 PN - w_1 MP$$

Since $\frac{w_1}{w_2} = \frac{O_2P}{O_1P} = \frac{PN}{MP}$

$$\Rightarrow v_s = (w_1 + w_2) QP$$

7.4 Inovolute Teeth:

An involute of circle is a plane curve generated by point on a tangent, which rolls on the circle without sliping or by a point on a taut string which in unwrapped from a reel.



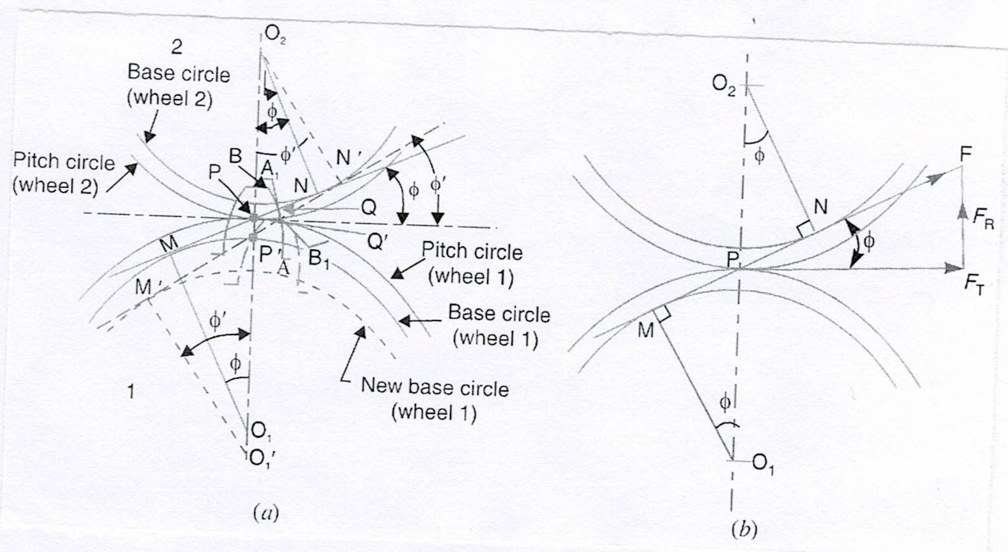
Let A be the starting point of the involut.

$AP_1 = P_1P_2 = P_2P_3$ etc. The tangents at P_1, P_2, P_3 etc. are drawn so as

$P_1A_1 = AP_1$, $P_2A_2 = AP_2$, $P_3A_3 = AP_3$.

Now let O_1 and O_2 be the fixed centres of the two base circle and the corresponding involutes AB and A_1B_1 be in contact at point Q.

Therefore the involute teeth satisfy the fundamental condition of constant velocity ratio.



$$\frac{O_1M}{O_2N} = \frac{O_1P}{O_2P} = \frac{\omega_2}{\omega_1}$$

$O_1M = O_1P \cos\phi$

$O_2N = O_2P \cos\phi$

$\therefore$ Centre distance between the base circles

$$O_1O_2 = O_1P + O_2P = \frac{O_1M}{\cos\phi} + \frac{O_2N}{\cos\phi} = \frac{O_1M + O_2N}{\cos\phi}$$

If F is the maximum tooth pressure

Tangential force, $F_T = F\cos\phi$

Radial force, $F_R = F\sin\phi$

Torque exerted on the gear shaft $= F_T \times r$

r: pitch circle radius of gear.

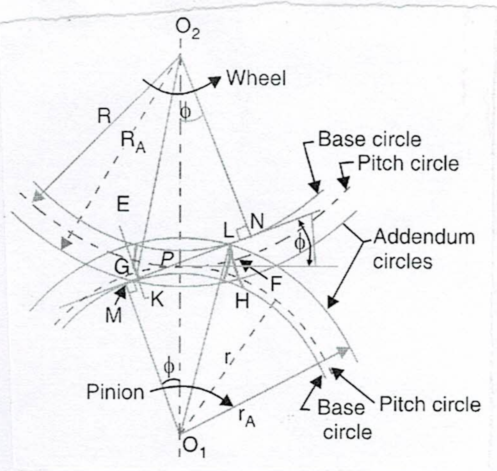
* effect of alternating the Centre distance on the velocity Ratio

If Centre of gear 1 is shifted from O_1 to O_1'

$$\frac{O_1'M'}{O_2N'} = \frac{O_1'P'}{O_2P'}$$

But $O_2N = O_2N'$ and $O_1M = O_1'M'$

$$\frac{O_1P}{O_2P} = \frac{O_1'P'}{O_2P'}$$



7.5. Length of path of contact:

Cosider a pinion driving the wheel

K is the intersection of the addendum circle of wheel and common tangent

L is the " " " " " pinion and common tangent

$\overline{KL}$ Path of Contact

$r_A = O_1L$ = Radius of addendum circle of pinion

$R_A = O_2K$ = " " " " wheel.

$r = O_1P$ = Radius of pitch circle of pinion

$R = O_2P$ = " " " " wheel.

$O_1M = O_1P\cos\phi = r\cos\phi$, $O_2N = O_2P\cos\phi = R\cos\phi$

$KN = \sqrt{(O_2K)^2 - (O_2N)^2} = \sqrt{(R_A)^2 - R^2\cos^2\phi}$ } from right angled

$PN = O_2P\sin\phi = R\sin\phi$ } triangle O_2KN.

Length of the path of approach,

$$\overline{KP} = \overline{KN} - \overline{PN} = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi$$

By similarly from right angle triangle O_1ML,

$ML = \sqrt{(O_1L)^2 - (O_1M)^2} = \sqrt{(r_a)^2 - r^2\cos^2\phi}$

$MP = O_1P\sin\phi = r\sin\phi$

Length path of recess

$$\overline{PL} = \overline{ML} - \overline{MP} = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi$$

∴ Length of the path of contact,

$$\overline{KL} = \overline{KP} + \overline{PL} = \sqrt{(R_A)^2 - R^2\cos^2\phi} + \sqrt{(r_A)^2 - r^2\cos^2\phi} - (R+r)\sin\phi$$

* Length of Arc of Contact

The Length of arc of approach $(\widehat{GP}) = \dfrac{\text{Length of path of approach}}{\cos\phi}$

$$= \frac{\overline{KP}}{\cos\phi}$$

Length of ~~arc~~ of recess $(\widehat{PH}) = \dfrac{\overline{PL}}{\cos\phi}$

Length of the arc of contact $= \widehat{GP} + \widehat{PH} = \dfrac{\overline{KP} + \overline{PL}}{\cos\phi} = \dfrac{\overline{KL}}{\cos\phi}$

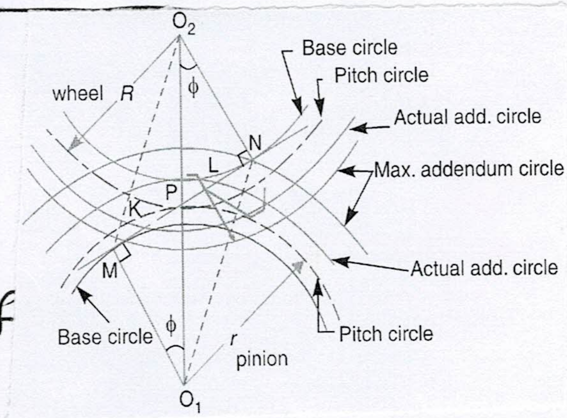
** Contact ratio (Number of pairs of Teeth in Contact)

Contact ratio $= \dfrac{\text{Length of the arc of Contact}}{P_c}$

P_c = Circular pitch $= \pi \cdot m$

7.5 Interference in Involute Gears:-

It is the phenomenon when the tip of tooth undercuts the root on its mating gear.



when interference is just avoided, the maximum length of path of contact is MN when the maximum addendum circles for pinion and wheel pass through the points of tangency N and M.

Max. length of path of approch, $\overline{MP} = r\sin\phi$

" length of path of recess, $\overline{PN} = R\sin\phi$

Max. Length of path of contact $= r\sin\phi + R\sin\phi = (r+R)\sin\phi$

Max. Length of arc of contact $= \dfrac{(r+R)\sin\phi}{\cos\phi} = (r+R)\tan\phi$

7.6 Minimum Number of Teeth on the pinion in order to Avoid Interference:–

The limiting condition reaches, when the addendum circles of pinion and wheel pass through points M and N.

Let, t = Number of teeth on the pinion,

T = " " " " wheel,

m = modul of the teeth

r = pitch circle radius of pinion $= m \cdot t/2$

G = Gear ratio $= T/t = R/r$

ϕ = pressure angle.

From triangle O_1NP.

$$(O_1N)^2 = (O_1P)^2 + (PN)^2 - 2\, O_1P \times PN \cos O_1PN$$

$$= r^2 + R^2 \sin^2\phi - 2rR \sin\phi \cos(90+\phi)$$

$$= r^2 + R^2 \sin^2\phi + 2r \cdot R \sin^2\phi$$

$$= r^2\left[1 + \frac{R^2 \sin^2\phi}{r^2} + \frac{2R\sin^2\phi}{r}\right] = r^2\left[1 + \frac{R}{r}\left(\frac{R}{r}+2\right)\sin^2\phi\right]$$

Limiting radius of the pinion addendum circle,

$$O_1N = r\sqrt{1 + \frac{R}{r}\left(\frac{R}{r}+2\right)\sin^2\phi} = \frac{m \cdot t}{2}\sqrt{1 + \frac{T}{t}\left[\frac{T}{t}+2\right]\sin^2\phi}$$

Let $A_p \cdot m$ = Addendum of the pinion, where A_p is a fraction by which the standrad addendum of one module for the pinion should be multiplied in order to avoid interference.

Addendum of the pinion $= O_1N - O_1P$

$$A_p \cdot m = \frac{m \cdot t}{2}\sqrt{1 + \frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi} - \frac{m \cdot t}{2}$$

$$\Rightarrow t = \frac{2A_p}{\sqrt{1 + G(G+2)\sin^2\phi} - 1}$$

where t the minimum number of teeth on the pinion.

** Minimum Number of Teeth on the wheel in order to Avoid Interference:-

T = Minimum number of teeth required on the wheel in order to avoid interference.

$A_W \cdot m$ = Addendum of the wheel, where A_W is a fraction by which the standard addendum for the wheel should multiplied.

From triangle O_2MP

$$(O_2M)^2 = (O_2P)^2 + (PM)^2 - 2 \times O_2P \times PM \cos O_2PM$$

$$= R^2 + r^2\sin^2\phi - 2R \cdot r\sin\phi\cos(90+\phi)$$

$$= R^2 + r^2\sin^2\phi + 2R \cdot r\sin^2\phi$$

$$= R^2\left[1 + \frac{r^2\sin^2\phi}{R^2} + \frac{2r\sin^2\phi}{R}\right] = R^2\left[1 + \frac{r}{R}\left(\frac{r}{R}+2\right)\sin^2\phi\right]$$

Limiting radius of wheel addendum circle:

$$O_2M = R\sqrt{1 + \frac{r}{R}\left(\frac{r}{R}+2\right)\sin^2\phi} = \frac{m \cdot T}{2}\sqrt{1 + \frac{t}{T}\left(\frac{t}{T}+2\right)\sin^2\phi}$$

Addendum of the wheel $= O_2M - O_2P$

$$A_W \cdot m = \frac{m \cdot T}{2}\sqrt{1 + \frac{t}{T}\left(\frac{t}{T}+2\right)\sin^2\phi} - \frac{m \cdot T}{2}$$

$$\therefore T = \frac{2A_W}{\sqrt{1 + \frac{t}{T}\left(\frac{t}{T}+2\right)\sin^2\phi} - 1} = \boxed{\frac{2A_W}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1}}$$

we can obtain the minimum number of teeth on pinion

Multiplying both sides by $\frac{t}{T}$

$$T \times \frac{t}{T} = \frac{2A_W \frac{t}{T}}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1}$$

$$\Rightarrow t = \frac{2A_W}{G\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1\right]}$$

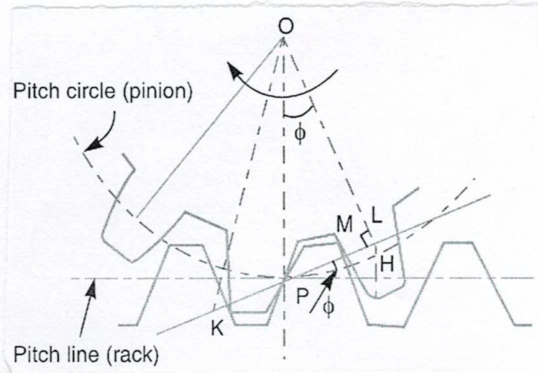
*** Minum Number of Teeth on a pinion for Involute Rack in order to Avoid Interference:—

Let t = Minimum number of teeth on the pinion

r = Pitch circle radius of the pinion $= m.t/2$

ϕ = pressure angle.

$A_R \cdot m$ = Addendum for rack



$$A_R \cdot m = LH = PL \sin\phi$$

$$= (OP \sin\phi) \sin\phi = OP \sin^2\phi$$

$$= r \sin^2\phi = \frac{m.t}{2} \times \sin^2\phi$$

$$t = \frac{2 A_R}{\sin^2\phi}$$

EX.1:– Two involute gears of 20° pressure angle are in mesh. The number of teeth on pinion is 20 and the gear ratio is 2. If the pitch expressed in module is 5mm and the pitch line speed is 1.2 m/s, assuming addendum as standard and equal to one module, find:
1- The angle turned through by pinion when one pair of teeth is in mesh 2- The maximum velocity of sliding.

Sol.

$$r = \frac{m \cdot t}{2} = \frac{5 \times 20}{2} = 50mm$$

$$R = \frac{m \cdot T}{2} = m \cdot G \cdot t/2 = \frac{2 \times 20 \times 5}{2} = 100mm$$

$$r_A = r + \text{Addendum} = 50 + 5 = 55mm$$

$$R_A = R + \text{Addendum} = 100 + 5 = 105mm$$

$$KP = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi \quad \text{path of approach.}$$

$$= \sqrt{(105)^2 - (100)^2(\cos 20)^2} - 100\sin 20 = 12.65\,mm$$

$$PL = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi \quad \text{path of recess}$$

$$= \sqrt{(55)^2 - (50)^2\cos^2 20} - 50\sin 20 = 11.5\,mm$$

$$KL = 12.65 + 11.5 = 24.15\,mm$$

$$\text{Arc of contact} = \frac{24.15}{\cos 20} = 25.7\,mm$$

$$\text{Angled turned by pinion} = \frac{\text{Arc of contact} \times 360°}{\text{Circumference of pinion}} = \frac{25.7 * 360°}{2\pi \times 50}$$

$$= 29.45°$$

$$\omega_1 = \frac{v}{r} = \frac{120}{5} = 24\,rad/s$$

$$\omega_2 = \frac{V}{R} = \frac{120}{10} = 12\,rad/s$$

Max. Velocity of sliding $v_s = (\omega_1 + \omega_2)KP$

$$= (12 + 24)\,12.65 = 455.4\,mm/s.$$

EX.2: Two mating gears have 20 and 40 involute teeth of module 10mm and 20° pressure angle. The addendum on each wheel is to be made of such a length that the line of contact on each side of the pitch point has half the maximum possible length. Determine the addendum height for each gear wheel, length of the path of contact, arc of contact and contact ratio.

Sol.

$$r = \frac{m.t}{2} = \frac{10 \times 20}{2} = 100mm$$

$$R = \frac{m.T}{2} = \frac{10 \times 40}{2} = 200mm$$

Path of approach, $KP = \frac{1}{2} MP$

or $\sqrt{(RA^2) - R^2\cos^2\phi} - R\sin\phi = \frac{r.\sin\phi}{2}$

$$\sqrt{(RA)^2 - (200)^2\cos^2 20} - 200\sin 20 = \frac{100 \times \sin 20}{2}$$

$\Rightarrow RA = 206.5mm$

$\therefore$ Addendum height for larger gear wheel $= RA - R = 206.5 - 200$
$= 6.5mm$

Path of recess, $PL = \frac{1}{2} PN$

$$\sqrt{(rA)^2 - r^2\cos^2\phi} - r\sin\phi = \frac{R\sin\phi}{2}$$

$$\sqrt{(rA)^2 - (100)^2\cos^2 20} = 100\sin 20 + 100\sin 20$$

$\Rightarrow rA = 106.2mm$

Addendum height for smaller gear wheel $= rA - r$
$= 106.2 - 100 = 6.2mm$

Length path of contact $= KP + PL = \frac{1}{2} MP + \frac{1}{2} PN$
$= \frac{(r+R)}{2}\sin\phi = 51.3mm$

Arc of contact $= \frac{\text{Path of contact}}{\cos\phi}$
$= \frac{51.3}{\cos 20} = 54.6mm$

Contact ratio $= \frac{\text{Arc of contact}}{P_c} = \frac{54.6}{\pi . m} = \frac{54.6}{\pi . 10} = 1.74 \simeq 2$

EX. 3:- A Pair of involute supr gears with 16° pressure angle and pitch of module 6mm is in mesh. The number of teeth on pinion is 16 and its rotational speed is 240 r.p.m. when the gear ratio is 1.75, find in order that the interference is just avoided 1. the addenda on pinion and gear wheel 2. the length of path of contact 3. the maximum velocity of sliding of teeth on on either side of the pitch point.

Sol. $G = \frac{T}{t} = 1.75 \Rightarrow T = 1.75 \times 16 = 28$

$$\text{Addendum on pinion} = \frac{m \cdot t}{2}\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi} - 1\right]$$

$$= \frac{6 \times 16}{2}\left[\sqrt{1+\frac{28}{16}\left(\frac{28}{16}+2\right)\sin^2 16} - 1\right]$$

$$= 10.76\text{mm}$$

$$\text{Addendum on wheel} = \frac{m \cdot T}{2}\left[\sqrt{1+\frac{t}{T}\left(\frac{t}{T}+2\right)\sin^2\phi} - 1\right]$$

$$= \frac{6 \times 28}{2}\left[\sqrt{1+\frac{16}{28}\left(\frac{16}{28}+2\sin^2 16\right.} - 1\right]$$

$$= 4.56\text{mm}.$$

$$R = \frac{m \cdot T}{2} = \frac{6 \times 28}{2} = 84\text{mm}, \quad r = \frac{m \cdot t}{2} = \frac{6 \times 16}{2} = 48\text{mm}$$

$$R_A = R + \text{Addendum of wheel} = 84 + 10.76 = 94.76\text{mm}$$

$$r_A = R + \text{Addendum of pinion} = 48 + 4.56 = 52.56\text{mm}$$

$$\text{Path of approach} = \sqrt{(R_A)^2 - R^2\cos^2\phi} - R\sin\phi = 26.45$$

$$\text{path of recess} = \sqrt{(r_A)^2 - r^2\cos^2\phi} - r\sin\phi = 11.94\text{mm}$$

$$\frac{w_1}{w_2} = \frac{T}{t} \Rightarrow w_2 = \frac{w_1}{1.75} = \frac{25.136}{1.75} = 14.28 \text{ rad/s}$$

$$V_{s\,max.L} = (w_1 + w_2)\,KP = (25.136 + 14.28)\,26.45 = 1043 \text{ mm/s}$$

$$V_{s.max.R} = (w_1 + w_2)\,PL = 471 \text{ mm/s. An.}$$

Theory of Machines - Third Stage

Q1/ A pinion having 20 involute teeth of module pitch 6mm rotates at 200 r.p.m. and transmits 1.5KW to a gear wheel having 50 teeth. The addendum on both the wheels is 1/4 of the circular pitch. The angle of obliquity is 20° Find (a) the length of the path of approach. (b) the length of the arc of approach, (c) the normal force between the teeth at an instant where there is only pair of teeth in contact.

[Ans. 13.27mm, 14.12 mm, 1193N]

Q2/ Two mating gears have 20 and 40 involute teeth of module 10mm and 20° pressure angle. If the addendum on each wheel is such that the path of contact is maximum and interference is just avoided, find the addendum for each gear wheel, path of contact arc of contact and contact ratio

[Ans. 14mm, 39mm, 102.6mm, 109.3mm, 4]

Q3/ A pinion with 24 involute teeth of 150mm of pitch circle diameter drives a rack. The addendum of pinion and rack is 6mm. Find the least pressure angle if interference avoid and find the length of arc of contact and the minimum number of teeth in contact at one time. [Ans. 16.8°, 40mm, 2 pairs of teeth]

Q4/ Two equal gear wheels of 360mm p.c.d. and 6 module are in mesh. If the pressure angle 20°, determine the maximum addendum necessary If there are always to be at least two pairs of teeth in contact. If the gear wheels rotate at 110 r.p.m. and 6KW is being transmitted, find the normal force between the teeth, assuming that the total force is divided equally between the two pairs of teeth. Neglect the friction

[Ans. 6.8mm, 1.54KN].

EX.4 :- A pinion of 20 involute teeth and 125 mm pitch circle diameter drives a rack. The addendum of both pinion and rack is 6.25 mm. What is the least pressure angle which can be used to aviod interference? what this pressure angle, find the length of the arc of contact and the minmum number of teeth in contact at a time.

Sol

Rack addendum, $LH = r\sin^2\phi \Rightarrow \sin^2\phi = \frac{6.25}{62.5} = 0.1$

$\Rightarrow \phi = 18.435°$

path of contact, $KL = \sqrt{(OK)^2 - (OL)^2}$

$= \sqrt{(OP+6.25)^2 - (OP\cos\phi)}$

$= 34.8\,mm$

Arc of contact $= \frac{\text{Path of contact}}{\cos\phi} = \frac{34.8}{\cos 18.435} = 36.68\,mm$

$P_c = \pi d / T = \frac{\pi \times 125}{20} = 19.64\,mm$

Contact ratio $= \frac{arc}{P_c} = \frac{36.68}{19.64} = 1.87 = 2$

Number of teeth in contact = 2

8. Gear Trains:-

It is a combination of two or more gears are made to mesh with each other to transmit power from one shaft to auther.

8.1. Simple Gear Train:-

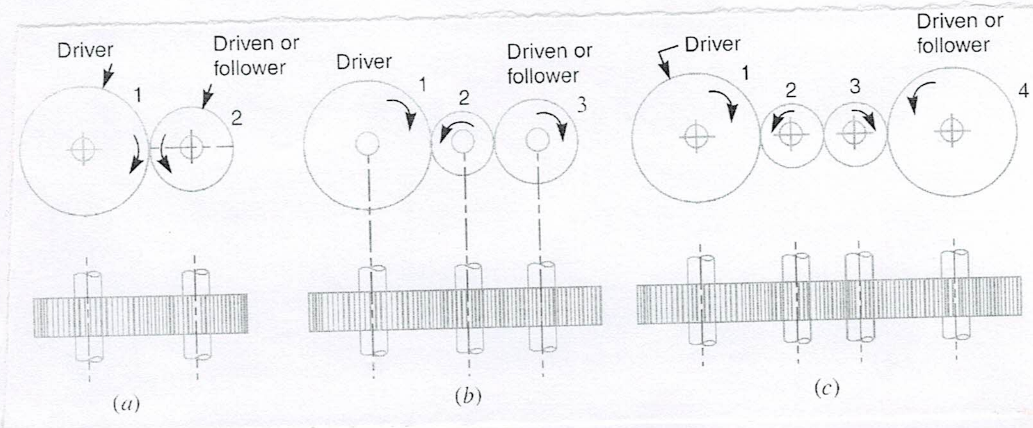


Fig.(1)

When there is only one gear on each shaft, as shown in above Fig., it is known as simple gear train.

If N = speed of gear, T = Number of teeth on gear.

$$\therefore \text{speed ratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

$$\text{Train value} = \frac{N_2}{N_1} = \frac{T_1}{T_2} = \frac{1}{\text{speed ratio}}$$

In Fig.(1-b) $\quad \frac{N_1}{N_2} = \frac{T_2}{T_1}, \quad \frac{N_2}{N_3} = \frac{T_3}{T_2}$

$$\frac{N_1}{N_2} \times \frac{N_2}{N_3} = \frac{T_2}{T_1} \times \frac{T_3}{T_2} \Rightarrow \frac{N_1}{N_3} = \frac{T_3}{T_1}$$

$$\text{Speed ratio} = \frac{\text{speed of driver}}{\text{speed of driven}} = \frac{\text{No. of teeth on driven}}{\text{No. of teeth on driver.}}$$

8.2. Compound Gear Train:–

when there are more than one gear on a shaft as shown in Fig.(2) it is called a compound gear train.

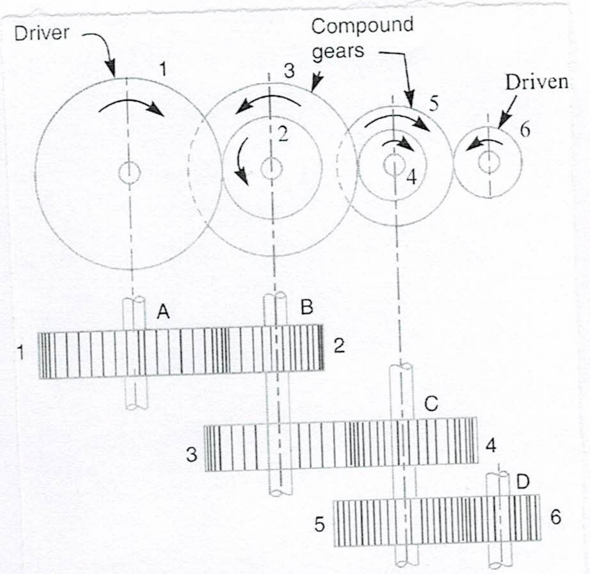


Fig.(2). Compound gear train.

$$\frac{N_1}{N_2} = \frac{T_2}{T_1}$$

$$\frac{N_3}{N_4} = \frac{T_4}{T_3}$$

$$\frac{N_5}{N_6} = \frac{T_6}{T_5}$$

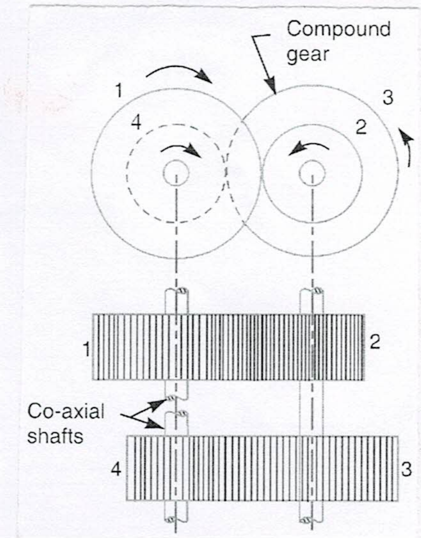
$$\frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

since $N_2 = N_3$ and $N_4 = N_5$

$$\Rightarrow \frac{N_1}{N_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5} = \text{speed ratio of the system}$$

8-3. Reverted Gear Train

When the axes of the first gear and the Last gear are co-axial then the gear train is known as reverted gear train.



$$r_1 + r_2 = r_3 + r_4$$

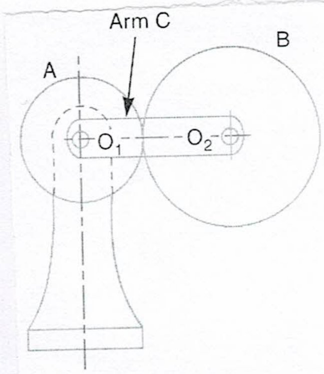
If m is equal to all

$$\Rightarrow T_1 + T_2 = T_3 + T_4$$

$$\frac{N_1}{N_4} = \frac{T_2 \times T_4}{T_1 \times T_3}$$

8.4. Epicyclic Gear Train:—

An epicyclic gear train shown in Fig. the axes of shafts, over which the gears are mounted, may move relative to a fixed axis.



Epicyclic gear train.

To find the velocity ratios of epicyclic gear train the table shows that.

Step No	Condition of motion	Revolutions of elements		
		Arm C	Gear A	Gear B
1.	Arm fixed - gear A +1 revolution anticlockwise	0	+1	$-\frac{T_A}{T_B}$
2.	Arm fixed - gear A rotates +x rev.	0	+x	$-x\frac{T_A}{T_B}$
3.	Add +y rev. to all elements	+y	+y	+y
4.	Total motion	+y	x+y	$y-x\times\frac{T_A}{T_B}$

If $N_A=0$

$\Rightarrow x=-y$

$$\frac{N_B}{N_C}=1+\frac{T_A}{T_B}$$

8.5 Compound Epicyclic Gear Train

A compound epicyclic gear train shown in Fig. consists of two co-axial shafts S_1 and S_2, an annulus gear A, the compound gear B-C, sun gear D and the arm H.

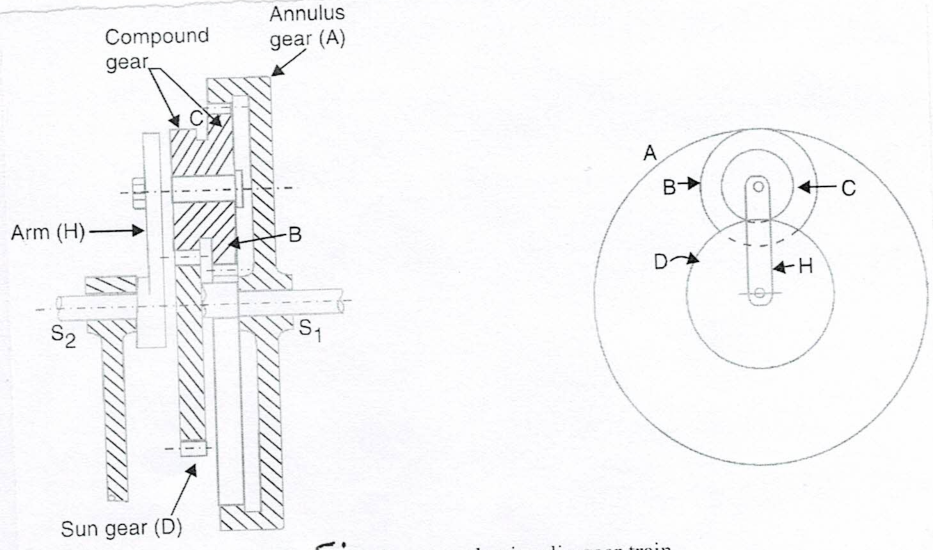


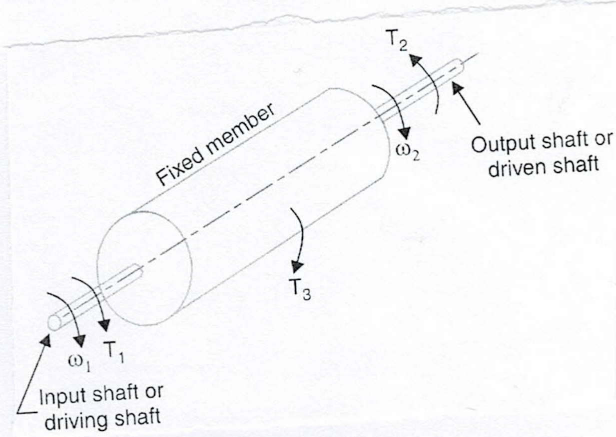
Fig. Compound epicyclic gear train.

Step No.	Conditions of motion	Revolutions of elements			
		Arm	Gear D	Compound Gear B-C	Gear A
1.	Arm fixed-gear D rotates through +1 revolution	0	+1	$-\frac{T_D}{T_C}$	$-\frac{T_D}{T_C} \times \frac{T_B}{T_A}$
2.	Arm fixed-g.D rotates +x r.	0	+x	$-x\frac{T_D}{T_C}$	$-x \times \frac{T_D}{T_C} \times \frac{T_B}{T_A}$
3.	Add +y r. to all elements	+y	+y	+y	+y
4.	Total motion	+y	x+y	$y-x\frac{T_D}{T_C}$	$y-x\frac{T_D}{T_C} \times \frac{T_B}{T_A}$

If the annulus gear A is rotated through one revolution anticlockwise with the arm fixed, then the compound gear rotates through T_A/T_B revolutions in the same sense and the sun gear D rotates through $-T_A/T_B \times T_C/T_D$.

8.6 Torques in Epicyclic Gear Trains:-

When the rotating parts of an epicyclic gear train have no angular acceleration, the gear train is kept in equilibrium.



$$T_1 + T_2 + T_3 = 0 \quad \text{——— (1)}$$

$$F_1 \cdot r_1 + F_2 \cdot r_2 + F_3 \cdot r_3 = 0 \quad \text{——— (2)}$$

$$T_1 w_1 + T_2 w_2 + T_3 w_3 = 0 \quad \text{——— (3)}$$

But, for a fixed member, $w_3 = 0$

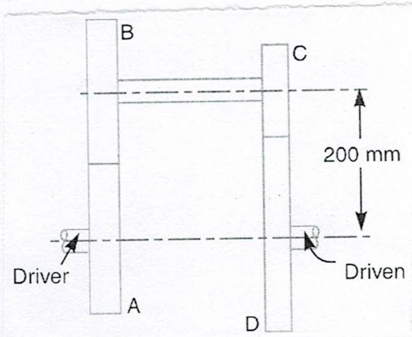
$$T_1 w_1 + T_2 w_2 = 0 \quad \text{——— (4)}$$

$$T_2 = -T_1 \times \frac{w_1}{w_2} \qquad \text{from (4)}$$

$$T_3 = -(T_1 + T_2) \qquad \text{from (1)}$$

$$= T_1 \left(\frac{w_1}{w_2} - 1 \right)$$

Exa. 1:- The speed ratio of reverted gear train shown in Fig. is to be 12. The module Pitch of gears A and B is 3.125 mm and of gears C and D is 2.5 mm. Calculate the suitable numbers of teeth for the gears. No gear is to have less than 24 teeth.



Sol

$$\frac{N_A}{N_B} = \frac{N_C}{N_D} = \sqrt{12} = 3.464$$

$$\frac{T_B}{T_A} = \frac{T_D}{T_C} = 3.464 \text{ ——— } (1)$$

$$x = r_A + r_B = r_C + r_D = 200 \text{ mm}$$

$$\frac{m_A T_A}{2} + \frac{m_B T_B}{2} = \frac{m_C T_C}{2} + \frac{m_D T_D}{2} = 200$$

$$3.125(T_A + T_B) = 2.5(T_C + T_D) = 400$$

$$T_A + T_B = 128 \text{ ——— } (2)$$

$$T_C + T_D = 160 \text{ ——— } (3)$$

From eqs. (1) and (2)

$T_A + 3.464\,T_A = 128$ or $T_A = 28.67$ say $\underline{28}$

$T_B = 128 - 28 = \underline{100}$

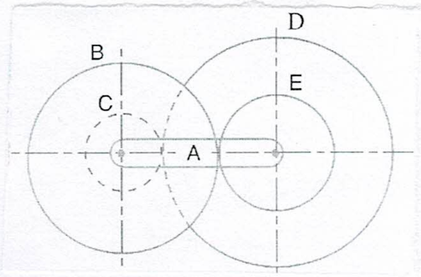
Similarly from eqs. (1) & (3)

$T_C + 3.464\,T_C = 160$ or $T_C = 35.84$ say 36

$T_D = 160 - 36 = 124$

$\therefore$ Speed ratio $\dfrac{N_A}{N_D} = \dfrac{T_B \times T_D}{T_A \times T_C} = 12.3$

Exa. 2:- In a reverted epicyclic gear train, the arm A carries two gears B and C and a compound gear D-E. The gear B meshes with gear E and the gear C meshes with gear D. The number of teeth on gears B, C and D are 75, 30 and 90 respectively. Find the speed and direction of gear C when gear B is fixed and the arm A makes 100 r.p.m. clockwise.



Sol. $d_B + d_E = d_C + d_D$

for the same module:

$T_B + T_E = T_C + T_D \quad \therefore T_E = T_C + T_D - T_B = 45$

The table of motions is drawn as follows:

Step No.	Conditions of motion	Revolutions of elements			
		Arm A	Compound gear D-E	Gear B	Gear C
1.	Arm fixed C.G. D-E rotated +1 rev.	0	+1	$-\frac{T_E}{T_B}$	$-\frac{T_D}{T_C}$
2.	Arm fixed C.G D-E rotated + x rev.	0	+x	$-x\frac{T_E}{T_B}$	$-x\frac{T_D}{T_C}$
3.	Add +y revolutions to all elements	+y	+y	+y	+y
4.	Total motion	+y	x+y	$y-x\frac{T_E}{T_B}$	$y-\frac{xT_D}{T_C}$

$\because$ Gear B is fixed $\Rightarrow y - x\frac{T_E}{T_B} = 0$

$y - x\frac{45}{75} = 0 \Rightarrow y - 0.6x = 0$ —— ①

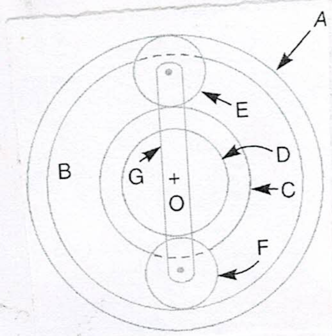
Arm A makes 100 r.p.m. Clockwise $\Rightarrow y = -100$ —— ②

Substituting $y = -100$ in eq. ①

$-100 - 0.6x = 0$ or $x = -166.67$ r.p.m.

$N_C = y - x \times \frac{T_D}{T_C} = -100 + 166.67 \times \frac{90}{30} = 400$ r.p.m anticlockwise.

Exa. 3:— In an epicyclic gear train, the internal wheels A and B and compound wheels C and D rotate independently about axis O. The wheels E and F rotate on pins fixed to the arm G. E gears with A and C and F gears with B and D. All the wheels have the same module and the number of teeth are: $T_C = 28$; $T_D = 26$; $T_E = T_F = 18$. 1. Sketch the arrangement; 2. Find the number of teeth on A and B; 3. If arm G makes 100 r.p.m clockwise and A is fixed find the speed of B 4. If the arm G makes 100 r.p.m. clockwise and wheel A makes 10 r.p.m. Counter clockwise find the speed of B.



Sol from the Geometry

$d_A = d_C + 2d_E$ and $d_B = d_D + 2d_F$

$T_A = T_C + 2T_E = 28 + 2 \times 18 = 64$

$T_B = T_D + 2T_F = 26 + 2 \times 18 = 62$

Table of motions

Step No.	Conditions of motion	Revolutions of elements					
		Arm G	wheel A	wheel E	C.W C–D	wheel F	wheel B
1.	Arm fixed wheel A +1 rev	0	+1	$+\frac{T_A}{T_E}$	$-\frac{T_A}{T_E} \times \frac{T_E}{T_C}$	$+\frac{T_A}{T_C} \times \frac{T_D}{T_F}$	$+\frac{T_A}{T_C} \times \frac{T_D}{T_F} \times \frac{T_F}{T_B}$
2.	Arm fixed wheel A +x rev	0	+x	$+x\frac{T_A}{T_E}$	$-x\frac{T_A}{T_C}$	$+x\frac{T_A}{T_C} \times \frac{T_D}{T_F}$	$+x\frac{T_A}{T_C} \times \frac{T_D}{T_B}$
3.	Add +y rev. for all elements	+y	+y	+y	+y	+y	+y
4.	Total motion	y	x+y	$y + x\frac{T_A}{T_y}$	$y - x\frac{T_A}{T_c}$	$y + x\frac{T_A}{T_c} \times \frac{T_D}{T_F}$	$y + x\frac{T_A}{T_c} \times \frac{T_D}{T_B}$

Since arm G makes 100 r.p.m clockwise $\Rightarrow y = -100$

Also the wheel A is fixed $\Rightarrow x + y = 0$ or: $x = -y = 100$ r.p.m

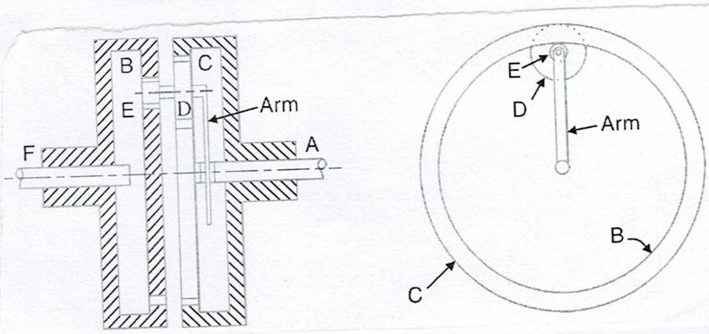
Speed of wheel $B = y + x \times \frac{T_A}{T_c} \times \frac{T_D}{T_c} = -100 + 100 \times \frac{64}{28} \times \frac{26}{62} = -4.2$ r.p.m

** If arm G makes 100 r.p.m Clockwise and wheel A makes 10 r.p.m. Counter clockwise: $y = -100$

$x + y = 10$ or $x = 10 - y = 10 + 100 = 110$

$\Rightarrow N_B = y + x \times \frac{T_A}{T_c} \times \frac{T_D}{T_B} = -100 + 110 \times \frac{64}{28} \times \frac{26}{62} = 5.4$ r.p.m.

Exa. 4:- An internal wheel B with 80 teeth is keyed to a shaft F. A fixed internal wheel C with 82 teeth is concentric with B. A compound wheel D–E; gears with the two internal wheels; D has 28 teeth and gears with C while E gears with B. The compound wheels revolve freely on a pin which projects from a disc keyed to a shaft A co-axial with F. If the wheels have the same pitch and the shaft A makes 800 r.p.m., what is the speed of the shaft F? Sketch the arrangement.



Sol. From the geometry

$$d_B = d_C - (d_D - d_E)$$

$$d_E = d_B + d_D - d_C$$

$$T_E = 80 + 28 - 82 = 26$$

Table of motion

Step No.	Conditions of motion	Revolutions of elements			
		Arm (shaft A)	wheel B (shaft F)	C.G D-E	Wheel C
1.	Arm fixed - wheel B rotated +1 rev.	0	+1	$+\frac{T_B}{T_E}$	$+\frac{T_B}{T_E} \times \frac{T_D}{T_C}$
2.	Arm fixed - wheel B rotated +x rev.	0	+x	$+x\frac{T_B}{T_E}$	$+x\frac{T_B}{T_E} \times \frac{T_D}{T_C}$
3.	Add +y rev. to all elements	+y	+y	+y	+y
4.	Total motion	+y	x+y	$y+x\frac{T_B}{T_E}$	$y+x\frac{T_B}{T_E} \times \frac{T_D}{T_C}$

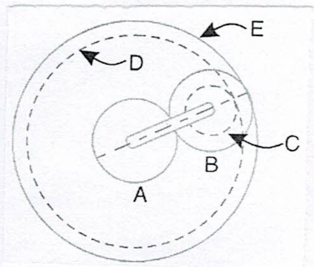
Since wheel C is fixed $\Rightarrow y + x\frac{T_B}{T_E} \times \frac{T_D}{T_C} = 0 \Rightarrow y + 1.05x = 0$

Shaft A makes 800 r.p.m. $\Rightarrow y = 800$

$\Rightarrow x = -762$

$N_F = N_B = x + y = -762 + 800 = 38$ r.p.m (anti clockwise).

Exa. 5:— An epicyclic gear train. pinion A has 15 teeth and is rigidly fixed to the motor shaft. The wheel B has 20 teeth gears with A and also with the annular fixed wheel E. Pinion C has 20 teeth and is integral with B. Gear C meshes with annular wheel D, which is keyed to the machine shaft. The arm rotates about the same shaft on which A is fixed and carries the compound wheel B, C. If the motor runs at 1000 r.p.m., find the speed of the machine and the torque exerted on the machine shaft, if the motor develops a torque of 100 N-m.



Sol.

$d_E = d_A + 2 d_B$, $d_D = d_E - (d_B - d_C)$

$\therefore T_E = 15 + 2 \times 20 = 55$, $T_D = 55 - (20 - 15) = 50$

Table of motions.

Step No.	Conditions of motion	Revolutions of elements				
		Arm	Pinion A	C.W B-C	wheel D	wheel E
1.	Arm fixed-pinion A rotated +1 rev.	0	+1	$-\frac{T_A}{T_B}$	$-\frac{T_A}{T_B} \times \frac{T_C}{T_D}$	$-\frac{T_A}{T_B} \times \frac{T_B}{T_E}$
2	Arm fixed-pinion A rotated +x rev.	0	+x	$-x \frac{T_A}{T_B}$	$-x \frac{T_A}{T_B} \times \frac{T_C}{T_D}$	$-x \times \frac{T_A}{T_E}$
3	Add +y rev. for all elements	+y	+y	+y	+y	+y
4.	Total motion	+y	x+y	$y - x \frac{T_A}{T_B}$	$y - x \frac{T_A}{T_B} \times \frac{T_C}{T_D}$	$y - x \frac{T_A}{T_E}$

$\therefore N_A = 1000$ r.p.m $\Rightarrow x + y = 1000$ —①

wheel is fixed $\Rightarrow y - x \times \frac{T_A}{T_E} = 0$ —② from eqs. ① & 2

$\Rightarrow y = 214$ & $x = 786$ r.p.m.

$N_D = N_{machine} \Rightarrow N_D = y - x \frac{T_A}{T_B} \times \frac{T_C}{T_D} = 214 - 786 \times \frac{15}{20} \times \frac{15}{50} = +37.15$ r.p.m.

From equillibrium $\Rightarrow$ Power developed by motor = Power developed by machine

$T_m \times W_m = T_{ma.} \times W_{ma.}$

$100 * W_A = T_{ma} \times W_{ma}$.

$T_{ma.} = 100 \frac{N_A}{N_D} = 100 \times \frac{1000}{37.15} = 2692$ N.m.

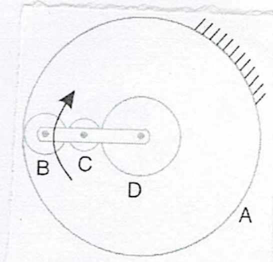
Theory of machines - Third Stage.

1. A compound train consists of six gears. The number of teeth on the gears are as follows:

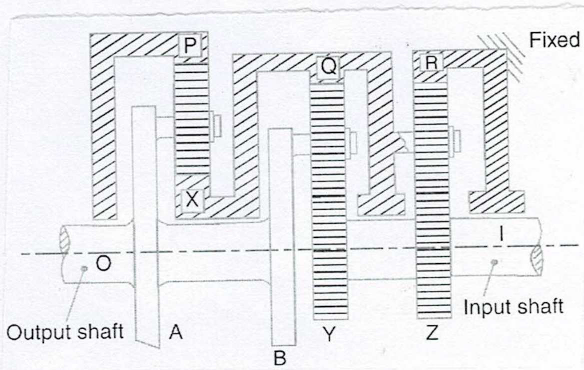
Gear :	A	B	C	D	E	F
No. of teeth:	60	40	50	25	30	24

The gears B and C are on the shaft while the gears D and E on another shaft. The gear A drives gear B, gear C drives gear D and gear E drives gear F. If the gear A transmits 1.5 KW at 100 r.p.m. and the gear train has the efficiency of 80 percent, find the torque on gear F. [Ans. 30.55 N.m]

Q_2: An epicyclic gear train, is composed of a fixed annular wheel A having 150 teeth. The wheel A is meshing with wheel B which drives wheel D through an idle wheel C, D being concentric with A. The wheels B and C are carried on an arm which revolves clockwise at 100 r.p.m. about the axis of A and D. If wheels B and D have 25 and 40 teeth respectively, find the number of teeth on C and the speed and sense of rotation of C. [Ans. 30; 600 r.p.m.]



Q_3: A compound epicyclic gear drive where I is the driving or input shaft and O is the driven or output shaft which carries two arms A and B rigidly fixed to it. The arms carry planet wheels which mesh with annular wheels P and Q and the sun wheels X and Y. The sun wheel X is a part of Q. Wheels Y and Z are fixed to the shaft I. Z engages with a plant wheel carried on Q and this plant wheel engages the fixed annular wheel R. The numbers of teeth on the wheels are: P = 114, Q = 120, R = 120, X = 36, Y = 24, Z = 30. If N_I = 1500 r.p.m clockwise and power = 7.5 kW find
1- Speed of shaft O and wheel P. 2. If efficiency 80% find torque on wheel R.



80 [Ans. 550 r.p.m, 66.84 N.m].

9. Belt Drives:—

It is used to transmit power from one shaft to anthor by means of pulleys which rotate at the same speed or different speeds.

9.1. Velocity Ratio of Belt Drive:-

If D_1 = Diameter of driver, D_2 = Diameter of follower;

L_1, L_2 = Length of the belt that passes over the driver and follower in one minute.

$\Rightarrow L_1 = \pi d_1 N_1$, $L_2 = \pi d_2 N_2$

since $L_1 = L_2$ in one minute.

$$\pi d_1 N_1 = \pi d_2 N_2$$

$$\Rightarrow \frac{N_2}{N_1} = \frac{d_1}{d_2} = \text{Velocity ratio.}$$

when thickness of belt is considered.

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}$$

9.2. Slip of Belt:-

It is a phenomenon which occurs when the frictional grip between the belt and the shaft becomes insufficient. This may also cause some forward motion of the belt without carrying the driven pulley with it. The result of the belt slipping is to reduce the velocity ratio of the system.

Let $S_1\%$, $S_2\%$ = slip between driver, follower and the belt.

∴ velocity of the belt passing over the driver per second.

$$v = \frac{\pi d_1 N_1}{60} - \frac{\pi d_1 N_1}{60} \times \frac{S_1}{100} = \frac{\pi d_1 N_1}{60}\left[1 - \frac{S_1}{100}\right] \quad \text{——(1)}$$

Velocity of the belt passing over the follower per second.

$$\frac{\pi d_2 \cdot N_2}{60} = v - v\frac{S_2}{100} = v\left[1 - \frac{S_2}{100}\right] \quad \text{——(2)}$$

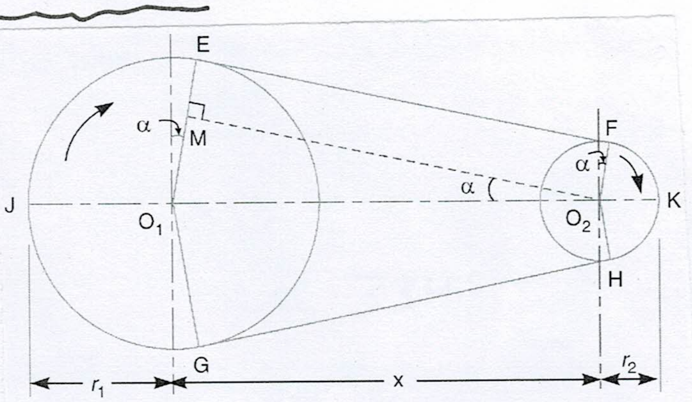
sub. eqs (1) in (2)

$$\frac{\pi d_2 N_2}{60} = \frac{\pi d_1 N_1}{60}\left[1-\frac{s_1}{100}\right]\left[1-\frac{s_2}{100}\right]$$

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}\left[1-\frac{s_1}{100}-\frac{s_2}{100}\right]$$

$$\frac{N_2}{N_1} = \left[1-\frac{s_1+s_2}{100}\right] = \frac{d_1}{d_2}\left[1-\frac{s}{100}\right]$$

9.3. Length of an open Belt Drive:-



Length of an open belt drive.

Let: r_1 & r_2 = Radii of larger and smaller pulleys

x = Distance between the Centers of two pullyes.

L = Total length of the belt.

$$L = \overset{\frown}{GJE} + EF + \overset{\frown}{FKH} + HG$$

$$= 2(\overset{\frown}{JE} + EF + \overset{\frown}{FK})$$

$$\sin\alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E - EM}{O_1O_2} = \frac{r_1 - r_2}{x} = \alpha$$

Since α is very small $\Rightarrow \sin\alpha = \alpha$ (in radians).

$$\overset{\frown}{JE} = r_1\left(\frac{\pi}{2}+\alpha\right), \quad \overset{\frown}{FK} = r_2\left[\frac{\pi}{2}-\alpha\right]$$

$$EF = \sqrt{(O_1O_2)^2 - (O_1M)^2} = \sqrt{x^2-(r_1-r_2)^2} = x\sqrt{1-\left[\frac{r_1-r_2}{x}\right]^2}$$

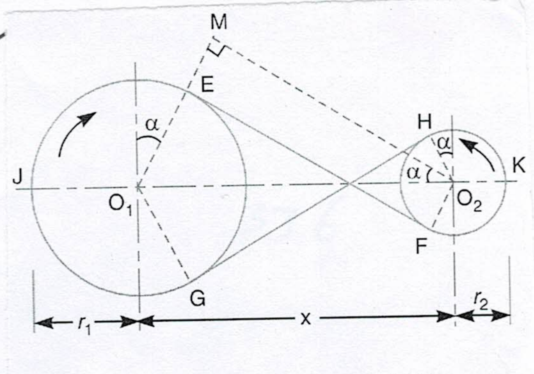
Expanding this equastion by binomial theorem.

$$\Rightarrow EF = x\left[1-\frac{1}{2}\left[\frac{r_2-r_2}{x}\right]^2+\cdots\right] = x - \frac{(r_1-r_2)^2}{2x}$$

$$\therefore L = 2\left[r_1\left(\frac{\pi}{2}+\alpha\right) + x - \frac{(r_1-r_2)^2}{2x} + r_2\left[\frac{\pi}{2}-\alpha\right]\right]$$

$$= \pi(r_1+r_2) + 2x + \frac{(r_1-r_2)^2}{x}$$

9.4. Length of a Cross Belt Drive:-



Length of a cross belt drive.

$$L = 2(\overset{\frown}{JE} + EF + \overset{\frown}{FK})$$

$$\sin\alpha = \alpha = \frac{r_1 + r_2}{x}$$

$$\overset{\frown}{JE} = r_1\left[\frac{\pi}{2} + \alpha\right], \quad \overset{\frown}{FK} = r_2\left[\frac{\pi}{2} + \alpha\right]$$

$$EF = \sqrt{x^2 - (r_1 + r_2)^2}$$

$$= x\sqrt{1 - \left(\frac{r_1 + r_2}{x}\right)^2} \text{ — Expanding by binomial theorem}$$

$$= x - \frac{(r_1 + r_2)^2}{2x}$$

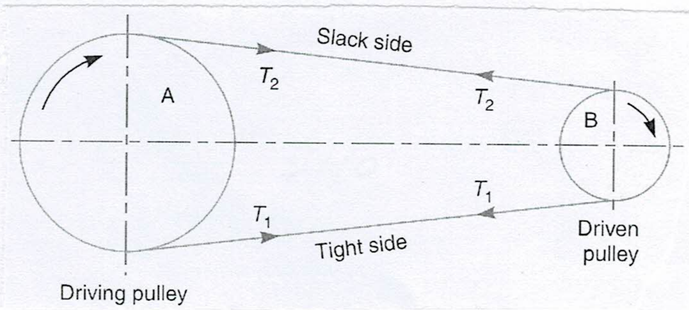
$$\therefore L = 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + x - \frac{(r_1 + r_2)^2}{2x} + r_2\left(\frac{\pi}{2} + \alpha\right)\right]$$

$$\Rightarrow L = \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}$$

9.5. Power Transmitted by a Belt:—

Let T_1, T_2 = Tension in the tight and slack side of the belt.

v = Velocity of belt.



Torque exerted on driving pulley $= (T_1 - T_2) r_1$.

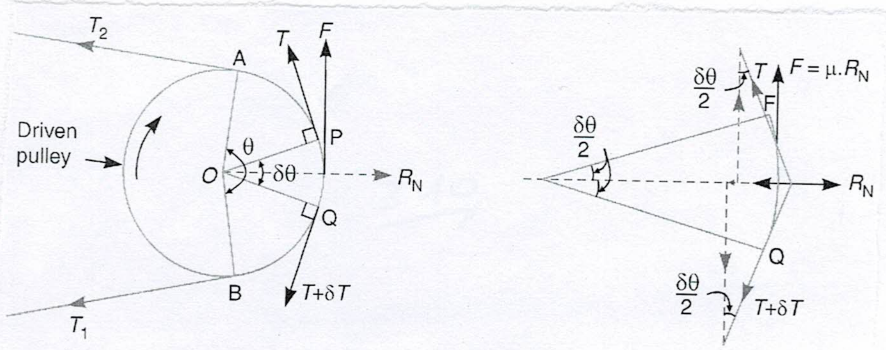
Torque exerted on driven pulley $= (T_1 - T_2) r_2$.

$\therefore$ Power transmitted $= (T_1 - T_2) v$.

9.6. Ratio of Driving Tensions For Flat Belt Drive:—

Let T_1, T_2 = Tension in the tight and slack sides.

θ = Angle of Contact in radians.



$$R_N = (T+\delta T)\sin\frac{\delta\theta}{2} + T\sin\frac{\delta\theta}{2} \quad \text{—(1)}$$ Horizontal forces.

since $\delta\theta$ is very Small $\Rightarrow \sin\frac{\delta\theta}{2} = \frac{\delta\theta}{2}$

$$R_N = (T+\delta T)\frac{\delta\theta}{2} + T \times \frac{\delta\theta}{2} = T\frac{\delta\theta}{2} + \frac{\delta T\,\delta\theta}{2} + \frac{T\,\delta\theta}{2} = T\,\delta\theta \quad \text{—(2)}$$

$$\mu R_N = (T+\delta T)\cos\frac{\delta\theta}{2} - T\cos\frac{\delta\theta}{2} \quad \text{—(3)}$$ vertical forces.

since $\delta\theta$ is very small $\Rightarrow \cos\frac{\delta\theta}{2} = 1$

$$\mu R_N = T + \delta T - T = \delta T \text{ or } R_N = \frac{\delta T}{\mu} \quad \text{—(4)}$$

From eqs. (4) & (2)

$T\,\delta\theta = \frac{\delta T}{\mu}$ or $\frac{\delta T}{T} = \mu\,\delta\theta$ Integration both Sides

$$\int_{T_2}^{T_1} \frac{\delta T}{T} = \mu \int_0^{\theta} \delta\theta \text{ or } \log_e\left(\frac{T_1}{T_2}\right) = \mu\theta \text{ or } \frac{T_1}{T_2} = e^{\mu\theta} \quad \text{—(5)}$$

eq. (5) Can be expressed in terms of Corresponding logarith to the base 10

$$\boxed{2.3\log\left(\frac{T_1}{T_2}\right) = \mu\theta}$$

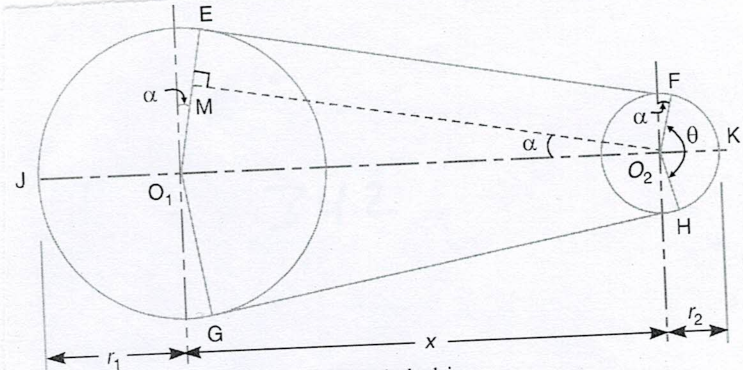
9.7. Angle of Contact:—

1- For the open belt drive

$$\sin\alpha = \frac{r_1 - r_2}{x}$$

$\therefore$ Angle of Contact

$$\theta = (180^\circ - 2\alpha)\frac{\pi}{180} \text{ rad.}$$

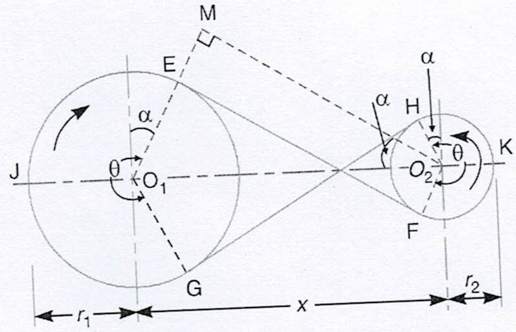


(a) Open belt drive.

2. For the crossed belt drive

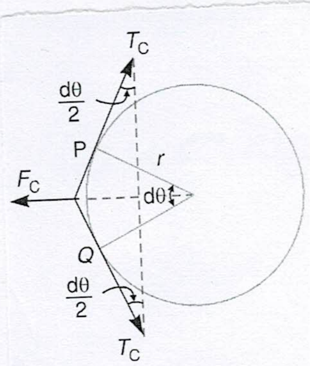
$$\sin\alpha = \frac{r_1 + r_2}{x}$$

$$\therefore \theta = (180^\circ + 2\alpha)\frac{\pi}{180} \text{ rad.}$$



(b) Crossed belt drive.

9.8. Centrifugal Tension:—



Consider a small portion PQ of the belt subtending an angle $d\theta$ the center of the pulley.

Let m = Mass of the belt per unit length.

v = Linear velocity of the belt.

r = Radius of the pulley which the belt runs.

T_c = Centrifugal tension acting tangentially at P and Q

Length of the belt $PQ = r\,d\theta$.

and mass of the belt $= m \cdot r\,d\theta$.

$$F_c = (m \cdot r \cdot d\theta)\frac{v^2}{r} = m\,d\theta \cdot v^2$$

$$T_c \sin\frac{d\theta}{2} + T_c \sin\frac{d\theta}{2} = F_c = m\,d\theta\,v^2 \quad \text{Horizontal forces.}$$

Let $\sin\frac{d\theta}{2} = \frac{d\theta}{2}$

$$\Rightarrow 2T_c\left(\frac{d\theta}{2}\right) = m\,d\theta\,v^2 \text{ or } T_c = mv^2$$

$$T_{t1} = T_1 + T_c \quad , \quad T_{t2} = T_2 + T_c$$

* Condition For the Transmission of Maximum power:-

$\because P = (T_1 - T_2)v$

$\because \frac{T_1}{T_2} = e^{\mu\theta}$ or $T_2 = \frac{T_1}{e^{\mu \cdot \theta}}$

$\therefore P = \left(T - \frac{T_1}{e^{\mu \cdot \theta}}\right) \cdot v = T_1\left(1 - \frac{1}{e^{\mu \cdot \theta}}\right) v = T_1 \, v \cdot C.$

$\because T_1 = T - T_c$

$\therefore P = (T - T_c) v \cdot C = (T - m v^2) v \cdot C = (T \cdot v - m v^3) C$

For maximum power, $\frac{dP}{dv} = 0$ or $\frac{d}{dv}(T \cdot v - m v^3) C = 0$

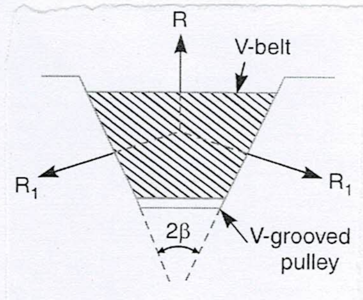
$\Rightarrow T - 3m v^2 = 0$

$T - 3T_c = 0 \Rightarrow T = 3T_c$

$T_1 = T - T_c = \frac{2T}{3}$

$v = \sqrt{\frac{T}{3m}}$

9.9 V- Belt Drive:-



Let R_1 = Normal reaction between the belt and sides of the groove.

R = Total reaction in the plane of the groove.

2β = Angle of the groove. μ = Coefficient of friction between the belt and sides of groove.

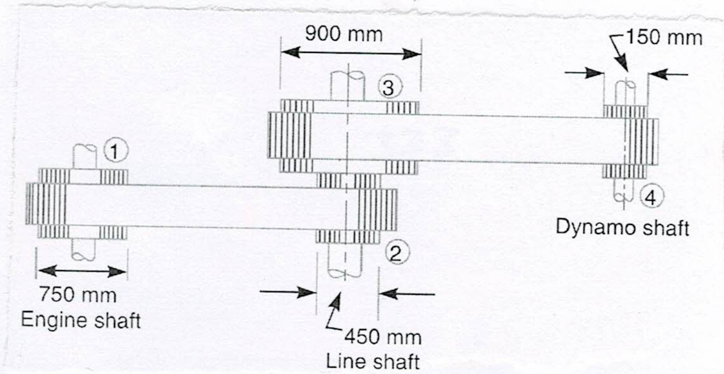
$R = R_1 \sin\beta + R_1 \sin\beta = 2 R_1 \sin\beta$

$\therefore R_1 = \frac{R}{2\sin\beta}$

Frictional force $= 2\mu R_1 = 2\mu \times \frac{R}{2\sin\beta} = \frac{\mu R}{\sin\beta} = \mu R \operatorname{cosec}\beta$

$\therefore 2.3 \log\left(\frac{T_1}{T_2}\right) = \mu \theta \operatorname{cosec}\beta.$

Exa. 1:- An engine, runnig at 150 r.p.m., drives a line shaft by means ofabelt. The engine pulley is 750 mm diameter and the pulley on the lineshaft being 450 mm. A 900 mm diameter pulley on the line shaft drives a 150 mm diameter pulley keyed to a dynamo shaft. Find the speed of the dynamo shaft when 1- there is no slip, and 2. thereis a slip of 2% at each drive.



Sol.

1- No slip:

Speed ratio

$$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4} \quad \text{or} \quad \frac{N_4}{150} = \frac{750 \times 900}{450 \times 150} = 10$$

2. Slip 2%:-

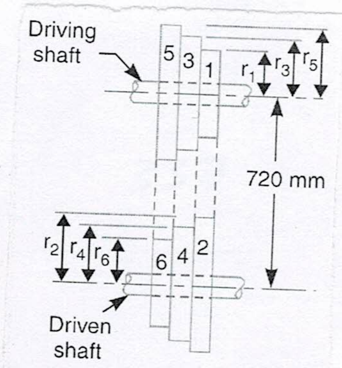
$$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}\left[1 - \frac{S_1}{100}\right]\left[1 - \frac{S_2}{100}\right]$$

$$\frac{N_4}{150} = \frac{750 \times 900}{450 \times 150}\left[1 - \frac{2}{100}\right]\left[1 - \frac{2}{100}\right] = 9.6$$

Exa.2 :- A shaft which rotates at a constant speed of 160 r.p.m is connected by belting to a parallel shaft 720 apart, which has to run at 60, 80 and 100 r.p.m. The smallest pulley on the driving shaft is 40 mm in radius. Determine the remaining radii of the two stepped pulleys for 1. a crossed belt, and 2- an open belt

Sol.

1. For a crossed belt:



$$\frac{N_2}{N_1} = \frac{r_1}{r_2} \Rightarrow r_2 = 40 * \frac{160}{60} = 106.7 \text{ mm.}$$

$$\frac{N_4}{N_3} = \frac{r_3}{r_4} \Rightarrow r_4 = 2 r_3$$

For a crossed belt.

$r_1 + r_2 = r_3 + r_4 = r_5 + r_6 = 40 + 106.7 = 146.7 \text{ mm}$

$r_3 + 2r_3 = 146.7 \Rightarrow r_3 = 48.9 \text{ mm}, r_4 = 97.8 \text{ mm}$

$r_6 = 1.6 r_5 \Rightarrow r_5 + 1.6 r_5 = 146.7 \Rightarrow r_5 = 56.4 \text{ mm}$

$r_6 = 90.2 \text{ mm}$

2. For an open belt:

$$\frac{N_2}{N_1} = \frac{r_1}{r_2} \Rightarrow r_2 = 106.7 \text{ mm}$$

$$L = \pi(r_1 + r_2) + \frac{(r_2 - r_1)^2}{x} + 2x$$

$= 1907$ mm

L constant for all pulleys

$$1907 = \pi(r_3 + r_4) + \frac{(r_4 - r_3)}{x} + 2x$$

$= 9.426 r_3 + 0.0014 r_3^2 + 1440$

$r_3 = 49.3 \text{ mm}, r_4 = 98.6 \text{ mm}$

also

$$1907 = \pi(r_5 + r_6) + \frac{(r_6 - r_5)^2}{x} + 2x$$

$\Rightarrow r_5 = 60 \text{ mm}$ and $r_6 = 1.6 r_5 = 96 \text{ mm.}$

Exa.3:- Two pulleys, one 450 mm diameter and the other 200 mm diameter are on parallel shafts 1.95 m apart and are connected by a crossed belt. Find the length of the belt required and the angle of contact between the belt and each pulley. What power can be transmitted by the belt when the larger pulley rotates at 200 r.p.m., if the maximum permissible tension in the belt is 1 KN, and the coefficient of friction between the belt and pulley is 0.25.

Sol.

$$v = \frac{\pi d_1 N_1}{60} = \frac{\pi \times 0.45 \times 200}{60} = 4.714 \text{ m/s}$$

$$L = \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(0.225 + 0.1 + 2 \times 1.95 + \frac{(0.225 + 0.1)^2}{1.95} = 4.975 \text{ m.}$$

$$\sin\alpha = \frac{r_1 + r_2}{x} = \frac{0.225 + 0.1}{1.95} \Rightarrow \alpha = 9.6^\circ$$

$$\theta = 180^\circ + 2\alpha = 199.2^\circ = 3.477 \text{ rad.}$$

$$2.3 \operatorname{Log}\left(\frac{T_1}{T_2}\right) = \mu\theta = 0.25 \times 3.477$$

$$\Rightarrow \frac{T_1}{T_2} = 2.387 \Rightarrow T_2 = \frac{1000}{2.387} = 419 \text{ N.}$$

$$\text{Power} = (T_1 - T_2)v = 2740 \text{ W.}$$

0.5m diameter, on parallel shafts 4m apart. The mass of the belt is 0.9 Kg/m and the maximum tension is not to exceed 2000N. The Coefficient of friction is 0.3. The 1.2m pulley, which is the driver, runs at 200 r.p.m. Due to the belt slip on one of the pulleys, the velocity of the driven shaft is only 450 r.p.m. Calculate the torque on each of the two shafts, the power transmitted, Powerlost in fricition and the effieiency of the drive.

Sol

$$v = \frac{\pi . d_1 . N_1}{60} = \frac{\pi \times 1.2 \times 200}{60} = 12.57 \text{ m/s}$$

Centrifugal tension, $T_c = m . v^2 = 0.9 (12.57)^2 = 142 N$

$$T_1 = T - T_c = 2000 - 142 = 1858 N$$

$$\sin\alpha = \frac{r_1 - r_2}{x} = \frac{0.6 - 0.25}{4} \Rightarrow \alpha = 5.02^\circ$$

$$\theta = 180^\circ - 2\alpha = 169.96^\circ = 2.967 \text{ rad.}$$

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu . \theta \Rightarrow T_2 = 762 N$$

Torque on the shafts of larger and smaller pulleys, T_L and T_s:

$$T_L = (T_1 - T_2) . r_1 = 657.6 \text{ N-m}$$

$$T_s = (T_1 - T_2) . r_2 = 274 \text{ N-m.}$$

Power transmitted, $P = (T_1 - T_2) v = (1858 - 762) 12.57 = 13780 W$

$$P_1 = \frac{T_L * 2\pi N_1}{60} = \frac{657.6 \times 2\pi \times 200}{60} = 13780 W$$ Input

$$P_2 = \frac{T_s * 2\pi N_2}{60} = \frac{274 \times 2\pi \times 450}{60} = 12910 W$$ output

Power Lost in friction $= P_1 - P_2 = 13.78 - 12.91 = 0.87 KW$

$$\eta = \frac{P_2}{P_1} = \frac{12.91}{13.78} = 93.7\%$$

Exa-5:- A belt drive Consiste of two V- belts in parallel, on grooved pulleys of the Same size. The angle of the groove is 30°. The cross-sectional area of each belt is 750 mm² and $\mu = 0.12$. The density of the belt material is 1.2 Mg/m³ and the maximum safe stress in the material is 7 Mpa. Calculate the Power that can be transmitted between pulleys 300 mm diameter rotating at 1500 r.p.m. Find also the shaft speed in r.p.m. at which the power transmitted would be maximum.

Sol.

$$v = \frac{\pi d N}{60} = \frac{\pi \times 0.3 \times 1500}{60} = 23.56 \text{ m/s}$$

$m =$ Area x Length x density $= 750 \times 10^{-6} \times 1 \times 1200 = 0.9$ Kg/m.

Centrifugal tension: $T_c = m.v^2 = 0.9 \times (23.56)^2 = 500\,N$

$$T_{max.} = \sigma_{max.} \times A = 7 \times 10^6 \times 750 \times 10^{-6} = 5250\ N$$

$T_1 = T - T_c = 5250 - 500 = 4750\,N$ tight side.

$\theta = 180° = \pi$ rad same size.

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu \theta \operatorname{Cosec} \beta, \quad \beta = 15°$$

$\Rightarrow T_2 = 1105\,N.$

$P = (T_1 - T_2) v \times 2 = 171.752$ KW

Let $N_1 =$ shaft speed, $v_1 =$ Belt speed.

For Maximum power:

$T_c = T/3$ or $m v_1^2 = T/3$

$0.9 (v_1)^2 = 5250/3 = 1750$

$v_1 = 44.1$ m/s

$$44.1 = \frac{\pi . d . N_1}{60} \Rightarrow N_1 = 2809 \text{ r.p.m.}$$